

## **Retrofitting Eagles Meadow Bridge with a Tuned Mass Damper**

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With a growing predilection for landmark bridge structures, particularly lightweight pedestrian crossings in town centres, comes a greater need to consider in detail the dynamic effects due to pedestrian loading. As structures become more slender relative to the structure mass (ratio of stiffness to mass reduces) and pedestrian loading becomes a more significant proportion of the total load, natural frequencies reduce and the acceleration response of the structure to pedestrian loading increases to the point where it can become disconcerting for the unsuspecting user. Once the natural frequency for such structures reduces to levels corresponding to normal walking frequencies, the bridge becomes susceptible to resonance and magnified acceleration responses. This may require modifications to the bridge to achieve an acceptable or imperceptible acceleration response to normal pedestrian usage. This paper reviews the design process that was undertaken to resolve the problem of the 'lively' response of a pedestrian footbridge at the Eagles Meadow development in Wrexham. Whilst it's acknowledged that there are sophisticated analytical techniques to assess such problems, it was found that a simplified approach, as presented below, could be used as a reasonably accurate means of assessing the effectiveness of possible modifications (by design provision or retrofitting), including the use of tuned mass dampers. As the bridge response was clearly due to excitation from pedestrians, the analysis includes modelling of the footfall dynamic loading in order to understand the response and evaluate possible solutions.

Eagles Meadow footbridge is one such landmark structure serving the prestigious town centre development in Wrexham. The bridge is an asymmetric cable-stayed structure with a curved steel and concrete composite deck and a single 'spike' pylon support, spanning approximately 13.5m and 34.5m over a road. The main span is supported by 3 pairs of cables, which are tied back with a pair of cables to an abutment at one end of the bridge.



Photo 1 – Eagles Meadow Bridge, Wrexham

Initial calculations indicated that the acceleration response would be within the limits given in BS 5400. However, when the bridge opened, it proved to be a little livelier than anticipated and the client requested that measures be taken to eliminate the vibration or reduce it to imperceptible levels. A site survey was carried out to determine the location and approximate magnitude of the maximum displacements and the loading arrangement that produced the maximum displacements. Maximum vertical displacements of approximately 10mm were recorded and some pedestrians were clearly disconcerted by the bridge acceleration effects. Not surprisingly, several people walking in step near the centre of the main span produced the worst effects, but it was also evident that with more people on the bridge in different locations, the response was significantly reduced. This indicated that several people walking at different locations and frequencies had a noticeable damping effect on the bridge response.

Structural models of the bridge were made to determine the principal natural modes of vibration, the natural frequencies and mode shapes using the finite element program STAADpro. This enabled the identification of the mode of vibration most likely to be associated with the resonance effects of pedestrian loading, which corresponded to alternating half waves in a near vertical plane in each span of the bridge (see Figure 1). The frequency could also be checked approximately by hand calculation using equation 2 below, assuming an effective mass of half the deck weight. In order to understand the dynamic behaviour of the bridge and, in particular, assess the acceleration response of the bridge, some further simplified hand calculations were carried out. Knowing the shape (Eigenvector) from the dynamic analysis (see Figure 1), each mode of vibration can be characterised as a Single Degree of Freedom (SDOF) structure (see Figure 2).

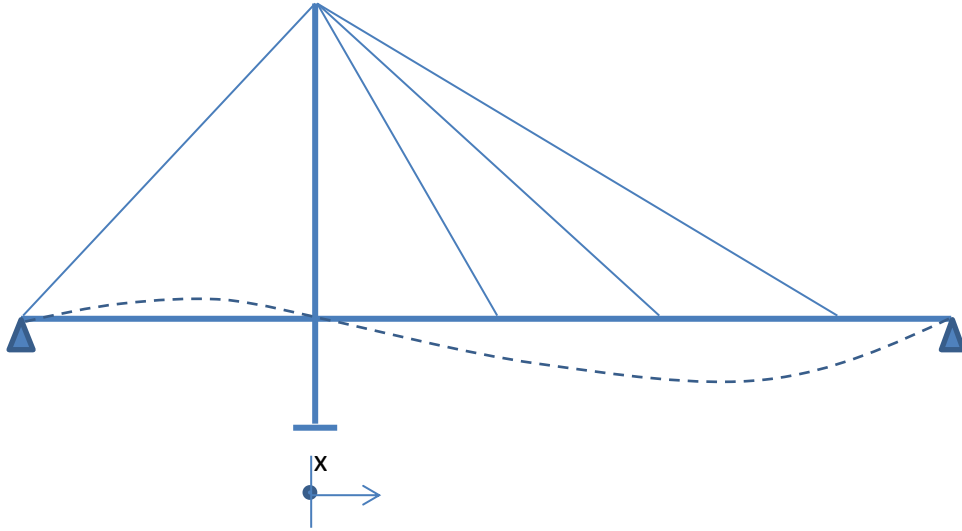


Figure 1 – Idealised Model of Bridge and Principal Mode of Vibration

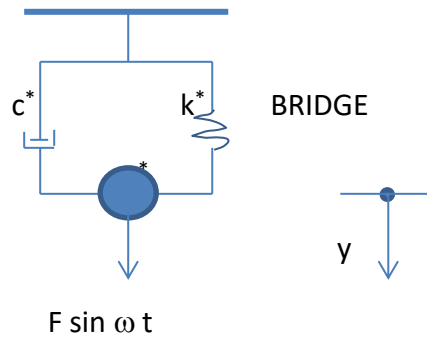


Figure 2 – Idealised Single Degree of Freedom Dynamic Model

The circular frequency of the SDOF system,  $\omega$ , is given as,

$$\omega = \sqrt{\frac{k^*}{m^*}} \quad \text{Equation 1}$$

The natural frequency,  $f$ , is then given by,

$$f = \frac{\omega}{2\pi} \quad \text{Equation 2}$$

Where  $k^*$  and  $m^*$  are the generalised stiffness (N/m) and mass of the structure (kg) for the particular mode of vibration of interest. The generalised stiffness and mass can be defined as follows (Clough and Penzien):

$$k^* = \int E I (x) f''(x)^2 dx + \sum k_i f(x_i)^2 \quad \text{Equation 3}$$

$$m^* = \int m (x) f(x)^2 dx + \sum m_i f(x_i)^2 \quad \text{Equation 4}$$

Where  $E$  is Young's Modulus,  $I$  is the moment of inertia of the bridge at position  $x$  along the bridge,  $f(x)$  is the shape function (Eigenvector) and  $f''(x)$  is the second differential of the shape function,  $m$  is the mass/metre,  $m_i$  is the discrete mass at  $x_i$  and  $k_i$  is the discrete spring stiffness at  $x_i$ . The maximum displacement ( $x$ ) should be taken as unity.

The shape function may be approximated to a sine wave ( $\sin \pi x/L$ ), which lends itself to easy differentiation and integration. Alternatively, as the shape function or mode shape is defined in the dynamic analysis as the Eigenvector for the particular mode, the integration for the generalised mass could be done by manual summation of the modal displacement along its length. As the frequency is known from the dynamic modal analysis and using the calculated generalised mass, the generalised stiffness,  $k^*$  could be determined from equation 1.

The damping in a bridge structure is typically taken as a generalised or average value rather than with discrete dashpots or other damping devices. In BS 5400 the damping was defined in terms of the logarithmic decrement,  $\delta$ , which can be approximately related to the critical damping ratio,  $\xi$ , as follows:

$$\xi = \frac{\delta}{2\pi} \quad \text{Equation 5}$$

The damping,  $c^*$ , can then be found from the following,

$$c^* = 2 \xi \omega m^* \quad \text{Equation 6}$$

The logarithmic decrement,  $\delta$ , was assumed to be 0.04 for a steel and concrete deck structure (refer to BS 5400 Part 2).

The dynamic pedestrian loading for a single person is defined in BS 5400 Part 2 as  $F \sin(2\pi f)t$  (or  $F \sin \omega t$ ) with a value of 180 N for  $F$ , which has been adopted for comparison purposes. However, recent studies (Willford and Young) suggest considerably higher dynamic amplitudes for the harmonic loading input for footfall, although this is likely to be due to several people walking perfectly in step, which is understood to be relatively infrequent (Barker). The sine wave form of loading is assumed to be a reasonable approximation of actual footfall loading and facilitates easy mathematical manipulation, when integrated or differentiated to solve equations of motion. More accurate periodic loading may be represented by Fourier series approximations or multiple harmonics, but this is likely to lead to considerably more complicated solutions.

The equation of motion (Beards) for the simplified single degree of freedom system is then:

$$m^* \frac{d^2 y}{dt^2} + c^* \frac{dy}{dt} + k^* y = F \sin \omega t \quad \text{Equation 7}$$

This second order differential equation can be solved as a summation of the solutions to the homogeneous and non-homogeneous parts, i.e.  $y = y_g + y_p$ . The homogeneous part

comprises the left hand side of equation 7 equated to zero. There is a standard general solution to the homogeneous part,  $y_g$ , to this type of differential equation, which takes the form,

$$y_g = e^{-\alpha t}(A \sin \beta t + B \cos \beta t)$$

However, as the initial displacement, velocity and acceleration are all zero, the constants A and B are also zero, thus making this part of the solution of no interest. This is understandable as it contains a decay function,  $e^{-\alpha t}$ , which would only be relevant to an initial discontinuous action. The particular solution,  $y_p$ , of the non-homogeneous part takes the following form,

$$y_p = (A \sin \omega t + B \cos \omega t)$$

Differentiating, substituting into equation 7 and equating sine terms (see Appendix),

$$y_p = -\frac{F}{c^* \omega} \cos \omega t \quad \text{Equation 8}$$

$$y_{p \max} = -\frac{F}{c^* \omega}$$

The frequency,  $\omega$ , is determined from idealised finite element modelling of the structure, simplified hand calculations as described above or by on-site measurement.

The bridge SDOF parameters have been estimated as follows:

$$m^* = 12000 \text{ kg}$$

$$k^* = 3000000 \text{ N/m}$$

$$c^* = 2250 \text{ kg/s}$$

This gave a maximum displacement of 9.3mm, which corresponded quite closely to the measured displacement.

Differentiating equation 8 twice gives the acceleration response as follows:

$$a = \frac{F \omega}{c^*} \cos \omega t$$

As the structure's natural frequency is a function of structure stiffness and mass, investigations were carried out using the computer model of the bridge to modify the stiffness of the bridge or to add mass in different locations on the structural models to achieve a sufficiently increased natural frequency and/or reduced acceleration. The effect of adding mass or modifying the stiffness could also be determined using equations 2 and 3 for discrete masses, springs or changes to the stiffness (E I) of the bridge.

However, as bridge stiffness and mass are linked (equation 2), it was found that significant changes to the dynamic response of the bridge were not possible without substantial modification of the structure. Adding mass in the deck tended to reduce the bridge frequency and didn't have any significant damping effects. Some consideration was also given to increasing the damping by modifying the parapets, which had been done on other bridges (Brown), but this was unlikely to be acceptable for the appearance and would be difficult to test and maintain. It was therefore concluded that by far the most pragmatic solution to reduce the acceleration response would be to fix a tuned mass damper to the deck and effectively damp out the unacceptable displacements and accelerations.

Tuned mass dampers contain a vertically oscillating dead weight, usually solid steel, supported on springs. In parallel to the springs, damping elements are arranged, which can be adjusted to the required damping,  $\xi$  – see Figure 3 below. The relative small mass of the tuned mass damper oscillates with a larger magnitude and just out of phase with the larger mass of the bridge deck, which has the effect of suppressing the motion of the bridge. The tuned mass damper can be bolted to the structure in the most effective location. The device would be custom designed to fit in the available space within or underneath the structure.

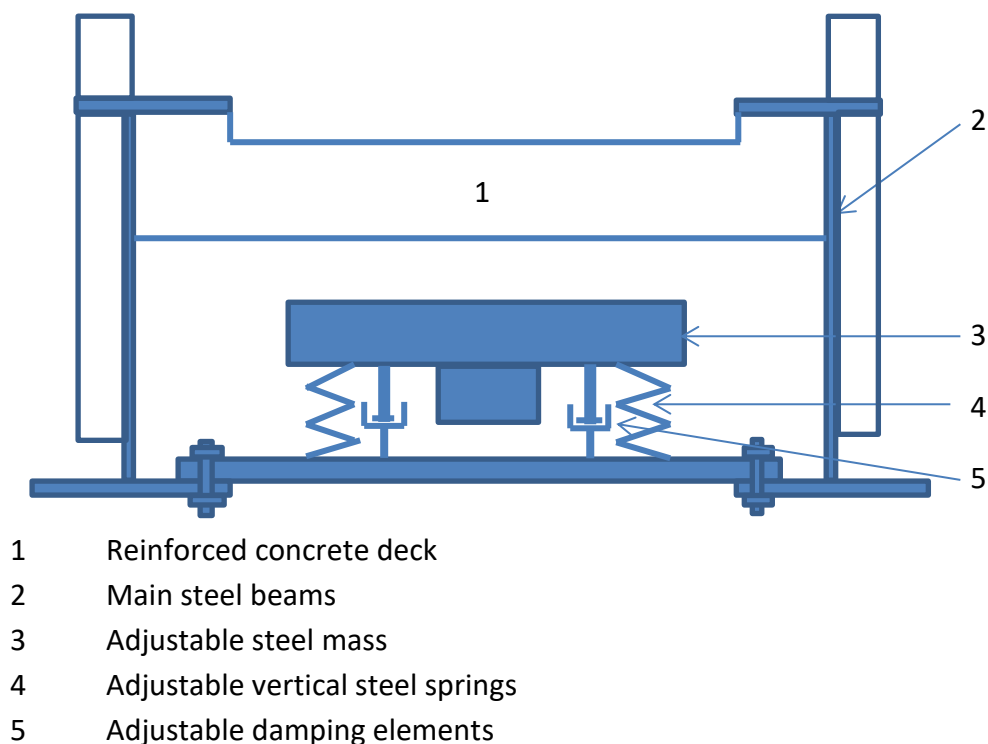


Figure 3 – Tuned mass damper details

The most effective location for a tuned mass damper would clearly be close to the location of maximum displacement indicated on Figure 1. It's possible that excessive displacements may occur at different locations due to other modes of vibration, in which case it may become necessary to add several tuned mass dampers to deal with each mode of vibration. However, it was found that a single tuned mass damper was sufficient on the Eagles

Meadow bridge. Extending the single degree of freedom simplification for the bridge, the effect of adding a tuned mass damper can be modelled as indicated in Figure 4 below.

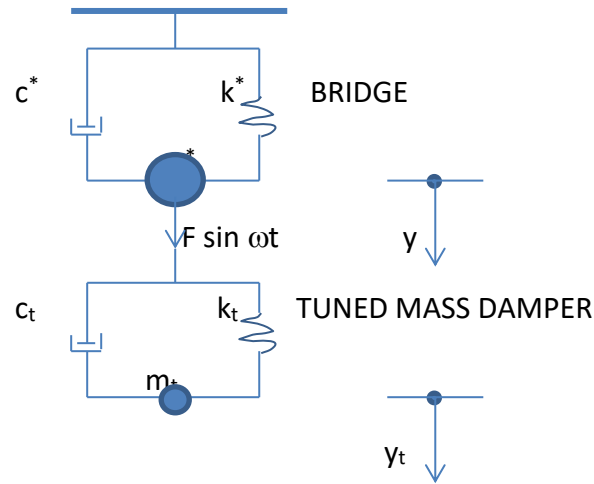


Figure 4 – Idealised Model of Bridge and Tuned Mass Damper

The equations of motion are then given by the following simultaneous differential equations (Beards),

$$m^* \frac{d^2 y}{dt^2} + c^* \frac{dy}{dt} + c_t \frac{d(y-y_t)}{dt} + k^* y + k_t (y - y_t) = F \sin \omega t \quad \text{Equation 9}$$

$$m_t \frac{d^2 y_t}{dt^2} + c_t \frac{d(y_t - y)}{dt} + k_t (y_t - y) = 0 \quad \text{Equation 10}$$

The solution (see Appendix) yields the following result for the displacement of the bridge,

$$y_p = M \sin \omega t + N \cos \omega t$$

Differentiating twice, gives the acceleration,

$$a = -M \omega^2 \sin \omega t - N \omega^2 \cos \omega t$$

Where,

$$M = \frac{(P Q - R S)}{(P^2 + R^2)}$$

$$N = \frac{(Q - P M)}{R}$$

$$P = m^* m_t \omega^4 - (m_t k^* + c^* c_t + m^* k_t + m_t k_t) \omega^2 + k_t k^*$$

$$Q = F (k_t - m_t \omega^2)$$

$$R = (m_t c^* + m_t c_t + m^* c_t) \omega^3 - (c_t k^* + c^* k_t) \omega$$

$$S = F (c_t \omega)$$

The most effective tuned frequency of the tuned mass damper is as close as possible to the natural frequency of the mode of vibration causing the excessive dynamic response. The frequency of the tuned mass damper is adjusted by changing the mass or modifying the springs. The damping can also be adjusted to optimise the effectiveness of the TMD. The bridge and tuned mass damper parameters were as follows:

$$m_t = 2000 \text{ kg}$$

$$k_t = 420500 \text{ N/m}$$

$$c_t = 4350 \text{ kg/s}$$

Substituting the estimated bridge and tuned mass damper parameters into the above equation gives a bridge deck displacement of approximately 0.1mm. Clearly, the tuned mass damper would have a dramatic effect on the dynamic response to pedestrian loading. Following on-site checking of the actual frequency of the bridge, the tuned mass damper was duly fitted to the bridge as close as possible to the point of maximum displacement under the deck between the two main steel beams – see Photo 2 below. Following installation of the tuned mass damper, the vertical movement of the bridge due to any forced pedestrian loading was barely perceptible.



Photo 2 – Tuned Mass Damper Installed Under Bridge Deck



## References

1. Dynamics of Structures, 2<sup>nd</sup> Edition (1993) by R.W. Clough and J. Penzien.
2. Structural Vibration Analysis, Modelling, Analysis and Damping of Vibrating Structures by C.F. Beards.
3. An Engineer's Approach to Dynamic Aspects of Bridge Design by C.W. Brown, Proceedings of Symposium on Dynamic Behaviour of Bridges, May 1977.
4. A Design Guide for Footfall Induced Vibration of Structures by M.R. Willford and P. Young (2006).
5. Footbridge Pedestrian Vibration Limits, Part 3: Background to Response Calculation by C. Barker, Footbridge 2005 – Second International Conference.

## Appendix

### Particular solution, $y_p$ , to differential equation 7

Differentiating,

$$\frac{dy}{dt} = A \omega \cos \omega t - B \omega \sin \omega t$$

$$\frac{d^2y}{dt^2} = -A \omega^2 \sin \omega t - B \omega^2 \cos \omega t$$

Substituting into equation 7,

$$-m^*A \omega^2 \sin \omega t - m^*B \omega^2 \cos \omega t + c^*A \omega \cos \omega t - c^*B \omega \sin \omega t + k^*A \sin \omega t + B k^* \cos \omega t = F \sin \omega t$$

Equating sine terms,

$$-m^*A \omega^2 - c^*B \omega + k^*A = F$$

Hence,

$$A(-m^* \omega^2 + k^*) - c^*B \omega = F \quad \text{but from equation 1, } (-m^* \omega^2 + k^*) = 0$$

So,

$$B = -\frac{F}{c^* \omega}$$

### Solution to simultaneous differential equations 9 and 10

Applying the 'D' operator notation for differential functions as follows:

$$D = \frac{d}{dt}, \quad D^2 = \frac{d^2}{dt^2}, \quad D^3 = \frac{d^3}{dt^3} \quad \text{etc}$$

Collecting the  $y$  and  $y_t$  terms, equations 9 and 10 can be re-written,

$$(m^*D^2 + c^*D + c_tD + k_t + k^*)y - (c_tD + k_t)y_t = F \sin \omega t \quad \text{Equation 9a}$$

$$-(c_tD + k_t)y + (m_tD^2 + c_tD + k_t)y_t = 0 \quad \text{Equation 10a}$$

Adding equations 9a and 10a,

$$(m^*D^2 + c^*D + k^*)y + (m_tD^2)y_t = F \sin \omega t \quad \text{Equation 11}$$

To eliminate  $y_t$ , 'operate' on equation 9a by multiplying by  $(m_tD^2)$  and equation 11 by  $(-c_tD - k_t)$ ,

$$(m^*m_tD^4 + m_t(c^* + c_t)D^3 + m_t(k_t + k^*)D^2)y - (m_t(c_tD + k_t)D^2)y_t = (m_tD^2)F \sin \omega t \quad \text{Equation 12}$$

$$(-c_tm^*D^3 - c_tc^*D^2 - c_tk^*D - k_tm^*D^2 - k_tc^*D - k_tk^*)y + (-c_tD - k_t)(m_tD^2)y_t = (-c_tD - k_t)F \sin \omega t \quad \text{Equation 13}$$

Subtract equation 13 from equation 12,

$$\{m^*m_tD^4 + (m_t(c^* + c_t) + m^*c_t)D^3 + (m_t(k_t + k^*) + c^*c_t + m^*k_t)D^2 + (c_tk^* + c^*k_t)D + k^*k_t\}y = \{(m_tD^2) + (c_tD + k_t)\}F \sin \omega t \quad \text{Equation 14}$$

Re-writing in conventional differential form and differentiating the right hand side,

$$A \frac{d^4y}{dt^4} + B \frac{d^3y}{dt^3} + C \frac{d^2y}{dt^2} + D \frac{dy}{dt} + E y = G \sin \omega t + H \cos \omega t \quad \text{Equation 15}$$

$$A = m^*m_t$$

$$B = m_t(c^* + c_t) + m^*c_t$$

$$C = m_t(k_t + k^*) + c^*c_t + m^*k_t$$

$$D = c_tk^* + c^*k_t$$

$$E = k^*k_t$$

$$G = F(k_t - m_t\omega^2)$$

$$H = F(c_t\omega)$$

The solution to equation 15, as before for equation 7, takes the form of a general solution plus a particular solution. The homogeneous or general part solves the transient initial value part for an initial imposed displacement, velocity or acceleration. However, as before, the initial conditions are all zero, so the general solution is of no interest and can be ignored. The behaviour of the bridge can therefore be determined from the solution to the non-homogeneous part, which takes the form,

$$y = M \sin \omega t + N \cos \omega t$$

The differentials in equation 15 can be evaluated, for example, as follows:

$$A \frac{d^4 y}{dt^4} = AM\omega^4 \sin \omega t + AN\omega^4 \cos \omega t$$

Substituting into equation 15, equating sine terms (equation 16), and cosine terms (equation 17),

$$(A\omega^4 - C\omega^2 + E)M + (B\omega^3 - D\omega)N = G \quad \text{Equation 16}$$

$$(-B\omega^3 + D\omega)M + (A\omega^4 - C\omega^2 + E)N = H \quad \text{Equation 17}$$

Multiply equation 16 by  $(A\omega^4 - C\omega^2 + E)$  and equation 17 by  $(-B\omega^3 + D\omega)$  and subtracting one from the other to eliminate N gives,

$$(A\omega^4 - C\omega^2 + E)^2 M + (B\omega^3 - D\omega)^2 M = (A\omega^4 - C\omega^2 + E)G - (B\omega^3 - D\omega)H$$

Therefore,

$$M = \{(A\omega^4 - C\omega^2 + E)G - (B\omega^3 - D\omega)H\} / \{(A\omega^4 - C\omega^2 + E)^2 + (B\omega^3 - D\omega)^2\}$$

And from equation 16,

$$N = \{G - (A\omega^4 - C\omega^2 + E)M\} / (B\omega^3 - D\omega)$$